MATHEMATICS

I. OVER-ALL AIM

The over-all aim of the Pre-University education is to inculcate mainly the following values among the students:

(i) General educational values viz cultural, disciplinary and practical or utilitarian values

(ii) Social, moral, aesthetic, intellectual and vocational values.

(iii) Patriotism. With reference to sciences, in particular mathematics, creating awareness about the contributions of Indians to Mathematics over centuries which are highlighted in relevant places.

II. SPECIFIC OBJECTIVES

(i) To create an aptitude for mathematics.

(ii) To create confidence in students for equipping themselves with mathematics needed for various branches of science, engineering and humanities.

(iii) To develop ability of reasoning and discrimination.

(iv) (a) Analysis of data, (b) Judging the relative relevance, adequacy and consistency of two sets of data.

(v) Drawing *valid* conclusion, consistent with rules of mathematical logic, from a set of given conditions or hypotheses.

(vi) Selection of the most appropriate among the available alternative methods to solve a problem.

(vii) Suggesting different methods (alternatives) for solving a given problem.

(viii) Skills of drawing or preparing graphs, charts and tables.

(ix) Assignments and projects with a view to inculcating independent and original thinking in solving mathematical problems and appreciating mathematical topics of great interest and value which are not covered in the formal syllabi.

(x) Enabling students to acquire *knowledge* of terms, symbols, formulae, definitions, concepts, processes and principles of mathematics.

SYLLABUS FOR I-YEAR P.U.C.

Chapter-I : Partial fractions

4 hours

Rational functions, proper and improper fractions, reduction of improper fractions as a sum of a polynomial and a proper fraction.

Rules of resolving a rational function into partial fractions in which denominator contains

(i) linear distinct factors, (ii) linear repeated factors, (iii) non-repeated non-factorizable quadratic factors [Problems limited to evaluation of three constants].

Expected learning outcomes

1. Identifies and classifies fractions as proper, improper
2. Selects the appropriate rule to find the partial fractions.
3. Uses methods of solving linear equations such as
   (i) making some terms zero and
   (ii) comparing the co-efficients of like terms.
4. Appreciates the technique of finding partial fractions.

Chapter-II : Logarithms

5 hours

(i) Definition of logarithm

(ii) Indices leading to logarithms and vice versa

(iii) Laws with proofs:

(a) $\log_a m + \log_a n = \log_a (mn)$

(b) $\log_a m - \log_a n = \log_a \left(\frac{m}{n}\right)$

(c) $\log_a m^n = n \log_a m$

(d) $\log_b m = \frac{\log_a m}{\log_a b}$ (change of base rule)

(iv) Common logarithm: Characteristic and mantissa; use of logarithmic tables; problems thereon.

Expected learning outcomes

1. Learns the conversion of index form to logarithmic form and vice versa.
2. Knows the limitations of bases.
3. Develops the art of using logarithmic tables
4. Appreciates the importance of laws of logarithms and logarithmic tables.

Chapter-III : Mathematical Induction **5 hours**

(i) Recapitulation of the n^{th} terms of an A.P. and a G.P. which are required to find the general term of the series.

(ii) Principle of mathematical induction – Proofs of

$$(a)\ \sum n = \frac{n(n+1)}{2}$$

$$(b)\ \sum n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$(c)\ \sum n^3 = \frac{n^2(n+1)^2}{4}$$

by mathematical induction

Sample problems on mathematical induction

Expected learning outcomes

1. Learns a new method of proof
2. Knows the limitations of this principle.

Chapter-IV: Summation of Finite Series **5 hours**

(i) Summation of series using Σn, Σn^2 and Σn^3

(ii) Arithmetico-geometric series

(iii) Method of differences – (when differences of successive terms are in A.P.)

(iv) By partial fractions

Expected learning outcomes

1. Learns methods of summation of finite series
2. Appreciates the techniques of reducing the given series into standard series.

Chapter -V: Theory of Equations **8 hours**

(i) Fundamental theorem of algebra: An n^{th} degree equation has n roots (without proof).

(ii) Solution of the equation $x^2+1=0$. Introducing square roots, cube roots and fourth roots of unity.

Expected learning outcomes

1. Appreciates various methods of solving equations
2. Learns how to find a root by inspection method and use it to find other roots
3. Understands the relations between roots and co-efficients and to use them to solve the equations.

(iii) Cubic and biquadratic equations, relations between the roots and the co-efficients. Solutions of cubic and biquadratic equations given certain conditions

(iv) Concept of synthetic division (without proof) and problems. Solution of equations by finding an integral root between −3 and +3 by inspection and then using synthetic division.

(v) Irrational and complex roots occur in conjugate pairs (without proof). Problems based on this result in solving cubic and biquadratic equations.

Chapter-VI : Binomial theorem **7 hours**

Permutations and Combinations

Recapitulation of ^nP_r and ^nC_r and proofs of

(i) general formulae for ^nP_r and ^nC_r

(ii) $^nC_r = {}^nC_{n-r}$

(iii) $^nC_{r-1} + {}^nC_r = {}^{n+1}C_r$

(1) Statement and proof of the Binomial theorem for a positive integral index by induction. Problems to find the middle term(s), terms independent of x and term containing a definite power of x.

(2) Binomial co-efficients – Proofs of

(1) $C_0 + C_1 + C_2 + \ldots\ldots\ldots\ldots + C_n = 2^n$

(2) $C_0 + C_2 + C_4 + \ldots\ldots\ldots\ldots = C_1 + C_3 + C_5 + \ldots\ldots\ldots\ldots 2^{n-1}$

Expected learning outcomes

1. Develops the ability of expansion when the index is very big.
2. Appreciates the method of finding a desired term without actual expansion.

Chapter -VII :Mathematical Logic **8 hours**

Proposition and truth values, connectives, their truth tables, inverse, converse, contrapositive of a proposition. Tautology and contradiction, logical equivalence – standard theorems. Examples from switching circuits, truth tables. Problems.

Expected learning outcomes

1. Learns decision making by using logic.
2. Appreciates the logical equivalences and the truth values of logical connectives.
3. Knows how to apply the concept of logic in electrical circuits.

Chapter-VIII : Sets, Relations and Functions **10 hours**

Recapitulation of sets, subsets, cardinality of a set ; Cartesian product of two sets.

Relations – reflexive, symmetric, transitive, equivalence and antisymmetric. Variables, function of a real variable, periodic functions. One-one (injective), onto (subjective), bijective, inverse and composite functions.

Expected learning outcomes

1. Recalls the language of set theory.
2. Learns the concepts like relation and functions and recognizes the difference between them.
3. Develops the skill of drawing graphs of selected functions.
4. Appreciates the terms like domain, co-domain and range of a function.

Types of Functions

Algebraic, trigonometric, hyperbolic, inverse trigonometric, exponential, logarthimic, parametric, $|x|$ and the greatest integer value function $[x]$.

Graphs of e^x, $\log x$, $|x|$ and $[x]$.

Chapter-IX: Trigonometry **35 hours**

1. **Measurement of angles and trigonometric functions** **25 hours**

(i) Radian measure – definition; proofs of

(a) Radian is constant (b) π radians $= 180°$ (c) $s = r\theta$ where θ is in radians (d) Area of a sector of a circle is given by $A = \frac{1}{2} r^2\theta$, where θ is in radians. Problems.

(ii) Trigonometric functions – definitions, trigonometric ratios of an acute angle. Trigonometric identities (with proofs) – Problems.

(iii) Trigonometric functions of standard angles. problems. Heights and distances – angle of elevation, angle of depression. Problems

Expected learning outcomes

1. Learns different units of measurement of angles and establishes relations between them.
2. Recognizes the importance of radian measure in finding the arc length and sectorial area of a circle.
3. Understands the relation between trigonometric functions.
4. Knows how to establish the relation between sides and angles of a triangle.
5. Calculates the values of trigonometric ratios of some angles and uses them to solve various problems such as heights and distances.

(iv) Trigonometric functions of allied angles, compound angles, multiple angles, submultiple angles and transformation formulae (with proofs) and problems.

(v) Graphs of $\sin x$, $\cos x$ and $\tan x$

2. Relations between sides and angles of triangle **10 hours**

(i) Sine rule, cosine rule, tangent rule; half-angle formulae; area of a triangle; projection rule (with proofs). Problems.

(ii) Solutions of triangles given (a) three sides, (b) two sides and the included angle, (c) two angles and a side and (d) two sides and the angle opposite to one of these sides. Problems.

Chapter-X: Analytical Geometry **27 hours**

1. Co-ordinate system **10 hours**

(i) Rectangular co-ordinate system in a plane (Cartesian)

(ii) Distance formula, section formula and mid-point formula, centroid of a triangle, area of a triangle – derivations and problems.

(iii) Locus of a point. Problems.

Expected learning outcomes

1. Learns to locate a point in a Cartesian plane.
2. Appreciates the conversion of geometrical conditions to algebraic equations.

2. Straight line **11 hours**

(i) Straight lines: Slope $m = \tan\theta$ of a line, where θ is the angle made by the line with the positive x-axis, slope of the line joining any two points, general equation of a line - derivations and problems.

(ii) Conditions for two lines to be (i) parallel, (ii) perpendicular. Problems.

(iii) Different forms of the equation of a straight line: (a) slope-point form, (b) slope-intercept form, (c) two points form, (d) intercept form and (e) normal form – derivations; problems.

(iv) Angle between two lines, point of intersection of two lines, condition for concurrency of three lines, length of the perpendicular from the origin and from any point to a line, equations of the internal and the external bisectors of the angle between two lines – derivations and problems.

3. Pair of straight lines **6 hours**

(i) Pair of lines: Homogeneous equation of second degree, general equation of second degree. Derivations of (1) condition for pair of lines, (2) condition for a pair of parallel lines, perpendicular lines and distance between the pair of parallel lines. (3) condition for a pair of coincident lines and (4) angle and point of intersection of a pair of lines.

Chapter – XI Limits and continuity **7 hours**

(1) Limit of a function – definition and algebra of limits.

(2) Standard limits (with proofs)

(i) $\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}$ (n rational)

(ii) $\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1$ (θ in radian)

and $\lim_{\theta\to 0}\frac{\tan\theta}{\theta}=1$ (θ in radian)

3) Statement of limits (without proofs):

(i) $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n=e$ (ii) $\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n=e^x$

(iii) $\lim_{x\to 0}(1+x)^{1/x}=e$ (iv) $\lim_{x\to 0}\frac{\log(1+x)}{x}=1$ (v) $\lim_{x\to 0}\frac{e^x-1}{x}=1$ (vi) $\lim_{x\to 0}\frac{a^x-1}{x}=\log_e a$

Expected learning outcomes

1. Understands the concept of left hand limit, right hand limit.
2. Learns different methods of evaluating limits.
3. Appreciates the concept of continuity.

Problems on limits

(4) Evaluation of limits which take the form $\lim_{x\to 0}\frac{f(x)}{g(x)}\left[\frac{0}{0}\text{form}\right], \lim_{n\to\infty}\frac{f(n)}{g(n)}\left[\frac{\infty}{\infty}\text{form}\right]$ where degree of $f(n)\le$ degree of $g(n)$. Problems.

(5) Continuity: Definitions of left-hand and right-hand limits and continuity.. Problems.

Chapter-XII: Elementary Graph Theory **4 hours**

Recapitulation of polyhedra and networks

(i) Definition of a graph and related terms like vertices, degree of a vertex, odd vertex, even vertex, edges, loop, multiple edges, u-v walk, trivial walk, closed walk, trail, path, closed path, cycle, even and odd cycles, cut vertex and bridges.

Expected learning outcomes

1. Learns a new branch of mathematics.
2. Develops the skill of drawing graphs.
3. Identifies different kinds of graphs.

(ii) Types of graphs : Finite graph, multiple graph, simple graph, (p,q) graph, null graph, complete graph, bipartite graph, complete graph, regular graph, complete graph, self-complementary graph, subgraph, supergraph, connected graph, Eulerian graph and trees.

(iii) The following theorems:

1) In a graph with p vertices and q edges $\sum_{i=1}^{p} \deg v_i = 2q$

2) In any graph the number of vertices of odd degree is even.

(iv) Definition of connected graphs, Eulerian graphs and trees – simple problems.

Assignments for I PUC

In the first year Pre-University course, each student has to submit 5 assignments — during the academic year — of the types mentioned below for deciding 5 marks, as prescribed by Department of PUE. Each assignment is to be valued for 10 marks and finally the total of the 5 assignments must be reduced for 5 marks.

Topics

Type-A

1) Logarithms – Some specific problems like finding the number of digits for a number having large power like $(3.2)^{98}$ etc., and also finding the number of zeros after the decimal point of a number like $(0.2)^{15}$, etc.

   Application of logarithmic tables in simplification of numerical compound expressions.

2) Problems on permutations and combinations
3) Binomial co-efficients – varieties of problems on summation involving them.
4) Theory of equations – formation of cubic and biquadratic equations, when the roots are given.
5) Methods of proof and disproof. Application of mathematical logic to examples from switching circuits.
6) Equation to locus of a point given some geometrical conditions. Problems on pair of lines.
7) Problems on heights and distances involving angles other than the standard angles (using mathematical tables). Relations between sides and angles of a triangle (harder problems).

Type-B

Familiarity with some mathematicians and their contributions:

1) Apastamba and Baudhayana (*Shulva sutras*)

2) Aryabhata I	3) Bhaskara I
4) Brahmagupta	5) Mahaviracharya
6) Bhaskaracharya II	7) Apollonius
8) Euclid	9) Pythagoras

Note: Out of the 5 assignments at least two must be from each of Type-A and Type-B. The teacher may also suggest any other topic under Type A and Type B for assignment, which will inspire the students.

Projects for I PUC

In the first year Pre-University course, each student has to submit 5 projects--during the academic year - of any of sample types mentioned below for deciding 5 marks, as prescribed by Department of PUE. Each project is to be valued for 10 marks and finally the total of 5 projects must be reduced for 5 marks.

Project 1: Graphs of the functions $\log x$, e^x, $1/x$, $|x|$, $[x]$, $\sin x$, $\cos x$, $\tan x$.

Project 2: Radian measure: Problems based on $s = r\theta$ and $A = \frac{1}{2} r^2 \theta$ with usual notations.

Project 3: Binomial Theorem:

i. Formation of Pascal's triangle and its properties

ii. To evaluate $(1.02)^6$, $(0.99)^5$, $(101)^7$, etc., using the Binomial theorem.

iii. To establish relation among Binomial co-efficients.

Project 4: To find the area of a triangle ABC given the vertices

i. Find points D and E which are collinear with C such that $\Delta ABC = \Delta ABD = \Delta ABE$

ii. Find points F and G which are non-collinear with C such that $\Delta ABC = \Delta ABF = \Delta ABG$.

Project 5: Relations and Functions

i. To construct relations on a set to show that reflexive, symmetric and transitive are independent.

ii. Given A and B

$f: A \rightarrow B$ is a function

a) Can you define *one-one* function?
If it exists, how many *one-one* functions can be defined.

b) Can you define *onto* function?
If it exists, how many such functions can be defined.

c) Can you construct bijective functions?
If it exists how many *bijective* functions exist?

Project 6: (1) Identify that the three given points are collinear.

(2) Find the equation of the common line.

(3) Find the slope of the line.

(4) Find the intercepts of the line

(5) Find the area made by the line with the co-ordinate axes

(6) Reduce the equation of a line to the normal form and find its distance from the origin.

Project 7: (1) Find the combined equation of the pair of lines $x = a, y = b$

(2) Find the independent lines for a given pair of lines (non-homogeneous form). Show the lines on a graph sheet and (i) measure the angle between them and (ii) find the point of intersection.

Also verify the results analytically.

Project 8 : (1) Preparing models of Polyhedra and verification of Euler's formula $F + V = E + 2$

(2) Study the problems :

a) Seven bridges problem
b) Chinese postman problem
c) Colouring a map (Four colour conjecture)
d) Utility problem

Project 9: Explain the principle used to resolve a given rational function into partial fractions.

(1) Why $\frac{3x^3+4x^2+2x+1}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$ has no solution ?

(2) Why $\frac{2x+1}{(x-1)(x+1)^2} = \frac{A}{x-1} + \frac{B}{(x+1)^2}$ has no solution ?

(3) Prove by mathematical induction that 6 divides $n(n+1)(2n+1)$

(4) Prove by mathematical induction that 5 divides 4^{2n}-1

(5) Prove by mathematical induction that the n^{th} term of $a, a+d, a+2d,$..........is $a+(n-1)d$.

Note: The teacher may also choose other topics, which will be of interest to students.

I year P.U.

Reference Text Books

1. Algebra by Hall and Knight
2. Algebra by Manickavachagam Pillai et al, S. Viswanath Printers Publishers Pvt. Ltd.
3. Theory ot Numbers by Manickavachagam Pillai et al
4. Coordinate Geometry by Shantinarayan
5. Coordinate Geometry by S.L. Loney
6. Higher Algebra by S. Barnard and J.M. Child, Macmillan India Ltd.
7. Plane Trigonometry by S.L. Loney
8. A Text Book of Trigonometry by S. Narayan, S.Viswanath Printers and Publishers Pvt. Ltd.
9. Graph Theory by V.R. Kulli, Vishwa International Publications, Gulbarga, 2000
10. Graph Theory by Frank Harary, Narosa Publishing House, New Delhi.

MATHEMATICS

Syllabus for II year PUC

Chapter – I Elements of Number Theory and Congruences 10 hours

(i) Divisibility – Definition and properties of divisibility; statement of division algorithm.

(ii) Greatest common divisor (GCD) of any two integers using Euclid's algorithm to find the GCD of any two integers. To express the GCD of two integers a and b as $ax + by$ for integers x and y. Problems

(iii) Relatively prime numbers, prime numbers and composite numbers, the number of positive divisors of a number and sum of all positive divisors of a number – statements of the formulae without proofs. Problems.

Expected learning outcomes

1. Understands the terms like divisibility and congruence.
2. Develops the skill of finding GCD using Euclid's algorithm.
3. Appreciates the application of properties of congruences in finding remainder and digit in units place.

(iv) Proofs of the following properties :

(1) the smallest divisor (>1) of an integer (> 1) is a prime number

(2) there are infinitely many primes

(3) if c and a are relatively prime and $c \mid ab$ then $c \mid b$

(4) if p is prime and $p \mid ab$ then $p \mid a$ or $p \mid b$

(5) if there exist integers x and y such that $ax+by=1$ then $(a,b) = 1$

(6) if $(a,b)=1$, $(a,c)=1$ then $(a,bc)=1$

(7) if p is prime and a is any integer then either $(p,a)=1$ or $p \mid a$

(8) the smallest positive divisor of a composite number a does not exceed $\sqrt{a}$

(v) Congruence modulo m –definition. Proofs of the following properties:

(1) "$\equiv$ mod m" is an equivalence relation

(2) $a \equiv b \pmod{m} \Rightarrow a \pm x \equiv b \pm x \pmod{m}$ and $ax \equiv bx \pmod{m}$

(3) If c is relatively prime to m and $ca \equiv cb \pmod{m}$ then $a \equiv b \pmod{m}$ – cancellation law.

(4) If $a \equiv b \pmod{m}$ and n is a positive divisor of m then $a \equiv b \pmod{n}$

(5) $a \equiv b (\text{mod } m) \Rightarrow a$ and b leave the same remainder when divided by m

(vi) Conditions for the existence of the solution of linear congruence $ax \equiv b(\text{mod } m)$ [statement only]. Problems on finding the solution of $ax \equiv b(\text{mod } m)$.

Chapter – II Matrices and Determinants **12 hours**

(i) Recapitulation of types of matrices; problems

(ii) Determinant of square matrix defined as mappings $\Delta: M(2,R) \rightarrow R$ and $\Delta: M(3,R) \rightarrow R$ Properties of determinants including $\Delta(AB) = \Delta(A)\,\Delta(B)$. Problems.

(iii) Minor and cofactor of an element of a square matrix, adjoint, singular and non-singular matrices, inverse of a matrix. Proof of $A(\text{Adj } A) = (\text{Adj } A)A = |A|\, I$ and hence the formula for A^{-1}. Problems.

(iv) Solution of a system of linear equations in two and three variables by (1) Matrix method, (2) Cramer's rule. Problems

Expected learning outcomes

1. Learns the properties of determinants and develops the skills of using them to evaluate a determinant without expansion.
2. Learns different methods of finding inverse of a square matrix.
3. Appreciates the matrix method of solving equations.
4. Appreciates the method of solving equation by Cramer's Rule.

(v) Characteristic equation and characteristic roots of a square matrix. Cayley-Hamilton theorem [statement only]. Verification of Cayley-Hamilton theorem for square matrices of order 2 only. Finding A^{-1} by Cayley-Hamilton theorem. Problems.

Chapter - III Groups **10 hours**

(i) Binary operation, algebraic structures. definitions of semigroup and group, Abelian group – examples from real and complex numbers. Finite and infinite groups, order of a group, composition tables, modular systems, modular groups, groups of matrices. Problems.

(ii) Square roots, cube roots and fourth roots of unity form abelian groups w.r.t. multiplication (with proof).

Expected learning outcomes

1. Learns the importance of Cayley's table.
2. Classifies sets with given operations as (i) semi-group (ii) group (iii) abelian group.
3. Appreciates the properties of groups
4. Understands the concept of subgroups and appreciates the sufficient condition for a subset to be a subgroup.

(iii) Proofs of the following properties :

(1) Identity element of a group is unique

(2) The inverse of an element in a group is unique.

(3) $(a^{-1})^{-1} = a, \forall\ a \in G$ where G is a group

(4) $(a * b)^{-1} = b^{-1} * a^{-1}$ in a group

(5) Left and right cancellation laws

(6) Solutions of $a*x=b$ and $y*a=b$ exist and are unique in a group.

(iv) Subgroups, proofs of necessary and sufficient conditions for a subgroup:

(a) A non-empty subset H of a group G is a sub group of G iff (1) $\forall\ a, b \in H, a*b \in H$ and (2) for each $a \in H, a^{-1} \in H$.

(b) A non-empty subset H of a group G is a subgroup of G iff $\forall a,b \in H, a*b^{-1} \in H$. Problems.

Chapter-IV : Vectors **10 hours**

(i) Definition of vector as a directed line segment, magnitude and direction of a vector, equal vectors, unit vector, position vector of point. Problems.

(ii) Two-and three-dimensional vectors as ordered pairs and ordered triplets respectively of real numbers, components of a vector, addition, subtraction, multiplication of a vector by a scalar. Problems

Expected learning outcomes

1. Learns the concept of vector and its representation as a directed line segment.
2. Distinguishes vector and scalar qualities
3. Appreciates the advantage of vector methods of establishing familiar results in trigonometry and geometry.

(iii) Position vector of the point dividing a given line segment in a given ratio.

(iv) Scalar (dot) product and vector (cross) product of two vectors.

(v) Section formula, mid-point formula and centroid

(vi) Direction cosines, direction ratios, proof of $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ and problems.

(vii) Application of dot and cross products to the area of a parallelogram, area of a triangle, orthogonal vectors and projection of one vector on another vector, problems.

(viii) Scalar triple product, vector triple product, volume of a parallelopiped; conditions for the coplanarity of 3 vectors and coplanarity of 4 points.

(ix) Proofs of the following results by the vector method:

(1) diagonals of parallelogram bisect each other, (2) angle in a semicircle is a right angle, and (3) medians of a triangle are concurrent; problems, (4) sine, cosine and projection rules, (5) proofs of (i) $\sin(A\pm B) = \sin A\cos B \pm \cos A\sin B$

(2) $\cos(A\pm B) = \cos A\cos B$ μ $\sin A\sin B$.

Chapter-V Circles **12 hours**

(i) Definition, equation of a circle with centre (0,0) and radius r and with centre (h,k) and radius r. Equation of a circle with (x_1, y_1) and (x_2,y_2) as the ends of a diameter, general equation of a circle, its centre and radius - derivations of all these. Problems.

(ii) Equation of the tangent to a circle – derivation; problems. Condition for a line $y=mx+c$ to be the tangent to the circle $x^2+y^2 = r^2$ - derivation, point of contact and problems.

(iii) Length of the tangent from an external point to a circle – derivation. Problems.

(iv) Power of a point, radical axis of two circles, Condition for a point to be inside or outside or on a circle - derivation and problems. Proof of the result "the radical axis of two circles is straight line perpendicular to the line joining their centres". Problems.

(v) Radical centre of a system of three circles – derivation. Problems.

(vi) Orthogonal circles – derivation of the condition. Problems.

Expected learning outcomes

1. Recognises equation of circles in various forms.
2. Appreciates the terms like power of a point with reference to a circle, Radical axis between two circles.
3. Recognises whether the circles touch each other or cut orthogonally.

Chapter VI Conic Sections: **13 hours**

Definition of a conic

1. Parabola **5 hours**

Equation of parabola using the focus directrix property (standard equation of parabola) in the form $y^2 = 4ax$; other forms of parabola (without derivation), equation of parabola in the parametric form; the latus rectum, ends and length of latus rectum. Equation of the tangent and normal to the parabola $y^2 = 4ax$ at a point (both in the cartesian form and the parametric form)

Expected learning outcomes

1. Understands the conics through their equations.
2. Appreciates the symmetry and acquires the skills of drawing the conics.

(1) Derivation of the condition for the line $y = mx + c$ to be a tangent to the parabola $y^2 = 4ax$ and the point of contact.

(2) The tangents drawn at the ends of a focal chord of a parabola intersect at right angles on the directrix - derivation

Problems.

2. Ellipse **4 hours**

Equation of ellipse using focus, directrix, and eccentricity - standard equation of ellipse in the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $(a > b)$ and other forms of ellipse (without derivations)

Equation of ellipse in the parametric form and auxiliary circle.

Latus rectum : ends and the length of latus rectum.

Equations of the tangent and the normal to the ellipse at a point (both in the cartesian form and the parametric form)

Derivations of the following:

(1) Condition for the line $y = mx + c$ to be a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at (x_1,y_1) and finding the point of contact

(2) Sum of the focal distances of any point on the ellipse is equal to the major axis

(3) The locus of the point of intersection of perpendicular tangents to an ellipse is a circle (director circle)

3. Hyperbola **4 hours**

Equation of hyperbola using focus, directrix and eccentricity - standard equation of hyperbola in the form $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. Conjugate hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$ and other forms of hyperbola (without derivations)

Equation of hyperbola in the parametric form and auxiliary circle.

The latus rectum ; ends and the length of latus rectum

Equations of the tangent and the normal to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at a point (both in the cartesian form and the parametric form)

Derivations fo the following results:

1) Condition for the line $y = mx + c$ to be tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and the point of contact.
2) Difference of the focal distances of any point on a hyperbola is equal to its transverse axis.
3) The locus of the point of intersection of perpendicular tangents to a hyperbola is a circle (director circle)
4) Asymptotes of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
5) Rectangular hyperbola
6) If e_1 and e_2 are eccentricities of a hyperbola and its conjugate then $\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1$

Trigonometry **18 hours**

Chapter VII Inverse trigonometric functions **6 hours**

(i) Definition of inverse trigonometric functions, their domain and range. Derivations of standard formulae. Problems

(ii) Solutions of inverse trigonometric equations. Problems

Expected learning outcomes

1. Appreciates the restrictions imposed on domain and co-domain for the existence of an inverse trigonometric function.
2. Relates one trigonometric function with the other.

Chapter-VIII General solutions of trigonometric equations **4 hours**

General solutions of $\sin x = k$, $\cos x = k$, $(-1 \le k \le 1)$, $\tan x = k$, $a\cos x + b\sin x = c$ – derivations. Problems.

Expected learning outcomes

1. Understands that solutions are infinite and understands the need for general solution.
2. Learns the skills of converting the given trigonometric equation in one of the standard forms and thereby finds the general solution.

Chapter-IX Complex numbers **8 hours**

(i) Definition of a complex number as an ordered pair, real and imaginary parts, modulus and amplitude of a complex number, equality of complex numbers, algebra of complex numbers, polar form of a complex number, Argand diagram, Exponential form of a complex number. Problems

(ii) De Moivre's theorem – statement and proof, method of finding square roots, cube roots and fourth roots of a complex number and their representation in the Argand diagram. Problems.

Expected learning outcomes

1. Learns three methods of writing a complex number and their inter-conversions.
2. Uses DeMoivre's theorem to find the n^{th} roots of a complex number.
3. Develops the skills of drawing Argand's diagram.

Calculus and Differential Equations **45 hours**

Chapter - X Differentiation **14 hours**

(i) Differentiability, Derivative of a function from first principles, Derivatives of sum and difference of functions, product of a constant and a function, constant, product of two functions, quotient of two functions from first principles. Derivatives of x^n, e^x, a^x, $\sin x$, $\cos x$, $\tan x$, $\operatorname{cosec} x$, $\sec x$, $\cot x$, $\log x$ from first principles. Problems.

Expected learning outcomes

1. Recognises the Rule to be used to differentiate the given function.
2. Uses differentiation to find maxima and minima and formation of differential equation.
3. Appreciates the use of differentiation in Physics and Geometry.

(ii) Derivatives of inverse trigonometric functions, hyperbolic and inverse hyperbolic functions.

(iii) Differentiation of composite functions – chain rule. Problems

(iv) Differentiation of inverse trigonometric functions by substitution. Problems

(v) Differentiation of implicit functions, parametric functions, a function w.r.t. another function, logarithmic differentiation. Problems.

(vi) Successive differentiation – problems upto second derivatives.

Chapter - XI Applications of derivatives **10 hours**

(i) Geometrical meaning of *dy/dx*, equations of tangent and normal, angle between two curves. Problems.

(ii) Subtangent and subnormal. Problems

(iii) Derivative as the rate measurer. Problems

(iv) Maxima and minima of a function of a single variable – second derivative test. Problems.

Chapter – XII Integration **8 hours**

(i) Statement of the fundamental theorem of integral calculus (without proof). Integration as the reverse process of differentiation. Standard formulae. Methods of integration, (1) substitution, (2) partial fractions, (3) integration by parts. Problems.

Expected learning outcomes

1. Recognises integration as an inverse of differentiation.
2. Learns different techniques of integration.
3. Finds the area under the curve using integration
4. Uses integration to solve differential equation.

(4) Problems on integrals of :

$$\frac{1}{a+b\cos x}, \frac{1}{a+b\sin x}, \frac{1}{a\cos x+b\sin x+c}, \frac{1}{a\sin^2 x+b\cos^2 x+c}$$

$$[f(x)]^n f^1(x), \frac{f^1(x)}{f(x)}, \frac{1}{\sqrt{a^2-x^2}}, \frac{1}{\sqrt{x^2-a^2}}, \frac{1}{\sqrt{a^2+x^2}},$$

$$\frac{1}{x\sqrt{x^2 \pm a^2}}, \frac{1}{a^2 \pm x^2}, \frac{1}{x^2-a^2}, \sqrt{a^2 \pm x^2}, \sqrt{x^2-a^2}, \frac{px+q}{ax^2+bx+c}, \frac{px+q}{\sqrt{ax^2+bx+c}}$$

$$\frac{p\cos x+q\sin x}{a\cos x+b\sin x}, e^x[f(x)+f^1(x)]$$

Chapter –XIII Definite Integrals and their applications **9 hours**

(i) Evaluation of definite integrals, properties of definite integrals, problems **5 hours**

(ii) Application of definite integrals: Area under a curve, area enclosed between two curves using definite integrals, standard areas like those of circle and ellipse. Problems **4 hours**

Chapter –XIV Differential Equations **4 hours**

Definitions of order and degree of a differential equation. Formation of a first order differential equation, problems. Solution of first order differential equations by the method of separation of variables, equations reducible to the variable separable form. General solution and particular solution. Problems.

Expected learning outcomes

1. Appreciates the method of formation of differential equation by eliminating arbitrary constant.
2. Recognises the order and degree of a differential equation.

*****

Assignments for II PUC

In the second year Pre-University course each student has to submit 5 assignments - during the academic year - of any type mentioned below for deciding 5 marks, as prescribed by the Department of PUE. Each assignment has to be valued for 10 marks and finally the total of the 5 assignments must be reduced for 5 marks.

Topics

Type–A

1) Application of vectors to mechanics such as work done by a force, moment of a force about a point, and about a line, moment of a couple.
2) (i) Application of congruences in finding the digit in the units place.
   (ii) to calculate the number of incongruent solutions of $ax \equiv b \pmod{m}$
3) Application of matrices : finding the *inverse* by Cayley – Hamilton theorem and also by reduction method. Also finding the *solution* of linear equations by reduction method.
4) Application of the theorem on subgroups.
5) Finding the length of the common chord of two intersecting circles.
6) Standard properties of conics applied to simple problems.
7) (i) Application of D'Moivre's theorem – finding n^{th} roots of a complex number.
   (ii) Expansion of $\sin 3\theta$ in terms of $\sin\theta$ and of $\cos 3\theta$ in terms of $\cos\theta$
   (iii) Expressing $\sin^3\theta$ and $\cos^3\theta$ in terms of sines and cosines of multiples of θ.
8) (i) Application of derivatives in finding maxima and minima of functions involving two dimensional figures.
   (ii) Formation of first and second order differential equations.
   (iii) Solution of $\frac{dy}{dx} + P(x) \quad y = Q(x)$
   (iv) Definite integral as a limit of a sum $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n} f\left(\frac{r}{n}\right) = \int_0^1 f(x)dx$

Type – B

Familiarity with some famous mathematicians and their contributions :

1) Leibnitz 2) Newton 3) Euler 4) Gauss 5)Cauchy

6) Descartes 7) Harish–Chandra 8)Srinivasa Ramanujan

9) Kerala mathematicians like

   (i) Madhava (ii) Paramesvara (iii) Neelakantha (iv) Achyuta Pisharathi

Note: Out of the 5 assignments at least two must be from each of Type – A and Type –B.

The teacher may also choose any other topic for assignment which will inspire the students.

Projects for II PUC

In the second year Pre–University course, each student has to submit 5 projects – during the academic year – of any type mentioned below for deciding 5 marks, as prescribed by the Department of PUE. Each project has to be valued for 10 marks and finally the total of 5 projects must be reduced for 5 marks.

Project 1: Groups: construct groups with (other than usual multiplication and addition) operations like

(1) Power set of a non-empty set X under the operation Δ given by

$A\Delta B = (A-B) \cup (B-A)$ and power set of X, $P(X) = \{ A | A \subseteq X\}$

Is $P(X)$ a group with respect to union ?

Is $P(X)$ a group with respect to intersection ?

Prove that $P(X)$ is a group with respect to Δ.

(2) Construct the set of permutations on $S=\{a,b,c\}$ and prove that it is a group with respect to multiplication of permutations.

Project 2: Properties of conics other than those which are in the syllabus

Project 3: Obtain the equations of (1) Coaxial system of circles, (2) Conjugate system of circles (3) Concentric circles and any other system, mathematically interesting, that you can think of.

Project 4: (1) Drawing a parabola using its equation

(2) Drawing a parabola using intersections of family of parallel lines and a family of concentric circles

(3) Draw a parabola as explained below:

Mark ten points say, at 1 cm intervals on two lines at right angles numbering them 1 to 10, join 1 to 10, 3 to 8 etc. and observe the diagram.

(4) Prepare models showing the intersection of a plane and a cone giving sections o the cone viz., circle, parabola, ellipse and hyperbola.

(5) Construction of parabolic reflector using the property that "If the lines draw from the focus meet the parabolic reflector they get reflected parallel to the ax of symmetry of the paraboloid".

Project 5: Problems using scalar product, vector product, scalar triple product, vector triple product and direction cosines.

(i) If α, β, γ are angles made by a vector with OX, OY, and OZ, then calculate

(a) $\sin^2 \alpha + \sin^2\beta + \sin^2\gamma$

(b) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$

(ii) Prove that

(a) $(\vec{p}.\hat{i})\hat{i}+(\vec{p}.\hat{j})\hat{j}+(\vec{p}.\hat{k})\hat{k}=\vec{p}$

(b) $(\vec{p}\times\hat{i})\times\hat{i}+(\vec{p}\times\hat{j})\times\hat{j}+(\vec{p}\times\hat{k})\times\hat{k}=-2\vec{p}$

(iii) To prove that the four given points are coplanar

(iv) Condition for $\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}\times\vec{b})\times\vec{c}$

Project 6:

(1) Differentiation from first principles of functions like $\sin x, e^{\sqrt{\sin x}}, \sin(\sqrt{x})$, etc.

(2) Application of derivatives in (a) maximizing the profit, (b) minimizing the cost

Project 7: Use properties of definite integrals in evaluating definite integrals such as

(a) $\int_0^2 \left[2^5-{}^5C_1 2^4 x+{}^5C_2 2^3 x^2-{}^5C_3 2^2 x^3+{}^5C_4 2x^4-x^5\right]dx$

(b) $\int_{-3}^{2}|x|dx$ (c) $\int_0^5 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}dx$ (d) $\int_{-\pi/6}^{\pi/6}\tan^7 x\ dx$ (e) $\int_0^{\pi}\cos^5 x\ dx$

Project 8:

(1) To find the number of divisors and the sum of divisors of a given number using

$$N=p_1^{\alpha_1}.p_2^{\alpha_2}........p_n^{\alpha_n}$$

$$T(N)=(1+\alpha_1)(1+\alpha_2).......(1+\alpha_n)$$

$$S(N)=\left(\frac{p_1^{\alpha_1}-1}{p_1-1}\right)\left(\frac{p_2^{\alpha_2}-1}{p_2-1}\right)........\left(\frac{p_n^{\alpha_n}-1}{p_n-1}\right)$$

(2) Euler's ϕ - function

To find the number of integers less than and relatively prime to N.

$$\phi(N)=N\left(1-\frac{1}{p_1}\right)\left(1-\frac{1}{p_2}\right)........\left(1-\frac{1}{p_n}\right)$$

(3) Vedic mathematics

Project 9: Application of matrices in economics

Project 10: (1) Recognise mathematical concepts in nature

(2) Write the geometrical equivalence for given algebraic equations

Note: The teacher may also choose any other topic for projects which will inspire the students.