Model Question Paper – 3

II P.U.C MATHEMATICS (35)

Time : 3 hours 15 minute **Max. Marks : 100**

Instructions :

(i) *The question paper has five parts namely A, B, C, D and E. Answer all the parts.*

(ii) *Use the graph sheet for the question on Linear programming in PART E.*

PART – A

Answer ALL the questions **$10 \times 1=10$**

1. Let * be the binary operation on N given by a*b=L.C.M of a and b, find 20*16
2. Write the domain of f(x)=$\tan^{-1} x$
3. Construct a 3x3 matrix A = (a_{ij}) Whose elements are given by $a_{ij} = \frac{i}{j}$
4. If $\begin{vmatrix} x & 2 \\ 3 & x \end{vmatrix} = \begin{vmatrix} x & 2 \\ -3 & -x \end{vmatrix}$, find the value of x
5. Differentiate log (cos e^x) w r t to x .
6. Evaluate :- $\int tan^2 2x.dx$
7. Find the angle between the two vectors $\vec{a}$ and $\vec{b}$ such that $|\vec{a}| =1$, $|\vec{b}| = 1$ and $\vec{a} . \vec{b}=1$
8. Find the equation of the plane having intercept 3 on the y axis and parallel to ZOX plane.
9. Define Linear objective function in linear programming problem
10. A fair die is rolled . Consider the events E={1,3,5} and F={2,3} , find P(E|F)

PART B

Answer any TEN questions: **$10 \times 2=20$**

11. Show that the relation R in the set of integers given by R = {(a, b) : 5 divides (a-b)} is symmetric and transitive.
12. If $\sin (\sin^{-1}\frac{1}{5} + \cos^{-1} x) = 1$, then find the value of x
13. Write the function $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ $x \neq 0$, in the simplest form.
14. If the area of the triangle with vertices (2.-6) and (5,4) and (k,4) is 35 sq. units . Find the value of k using determinant method.
15. Find the derivative of $\sqrt{x} + \sqrt{y} = 9$ at (4,9)
16. If y= $\log_7(\log x)$, find $\frac{dy}{dx}$
17. Evaluate $\int \frac{3x^2}{1+x^6} dx$

18. Evaluate $\int e^x \left(\frac{x-1}{x^2}\right)$dx

19. Find the slope of the tangent to the curve y=$x^3 - 3x + 2$ at the point whose x co ordinate is 3

20. Find the order and degree of the differential equation

$$xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx}=0$$

21. Show that the points A($2\hat{\imath}$-$\hat{\jmath}$+$\widehat{k}$), B(i -3$\hat{\jmath}$-5$\hat{k}$) and C(3 i -4$\hat{\jmath}$-4$\hat{k}$) are the vertices of a right angled triangle.

22. If $\vec{a}$ is a unit vector and $(\vec{x} - \vec{a}).(\vec{x} + \vec{a})$=15, then find $|\vec{x}|$

23. Find the distance of the point (2,3,-5) from the plane $\vec{r}.(\hat{\imath} + 2\hat{\jmath} - 2\hat{k})$=9

24. If the probability distribution of X is

X	0	1	2	3	4
P(X)	0.1	K	2k	2k	k

Find the value of k.

PART C

Answer any TEN questions: **10 $\times$ 3=30**

25. Verify whether the function f: R – {3} → R – {1}, defined by f(x) = $\frac{x-2}{x-3}$ is one-one and onto or not. Give reason.

26. Find the value of $tan\frac{1}{2}\left[\sin^{-1}\left(\frac{2x}{1+x^2}\right) + \cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right]$ |x|<1 , y>0 and xy<1

27. If A=$\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$, find A^{-1} by elementary operations.

28. If y= $\tan^{-1}\left(\frac{sinx}{1+cosx}\right)$ then prove that $\frac{dy}{dx} = \frac{1}{2}$

29. Differentiate $x^{sinx} + (sin\,x)^{cos\,x}$ w. r. t. x

30. Find the absolute maximum value and the absolute minimum value of the function f(x)=sinx+cosx, x∈[0,π]

31. Evaluate $\int \frac{dx}{x(x^n+1)}$

32. Evaluate $\int e^x sinx\, dx$

33. Find the area of the circle x^2+y^2 =4 bounded by the lines x=0 and x=2 which is lying in the first quadrant.

34. In a bank, principal "P" increases continuously at the rate of 5% per year. Find the principal interest of time t.

35. Find a vector perpendicular to each of the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ where $\vec{a} = 3\hat{\imath}$ +2$\hat{\jmath}$+2$\hat{k}$ and $\vec{b} = \hat{\imath}$ +2$\hat{\jmath}$-2$\hat{k}$

36. If $\vec{a}$=$-4\hat{\imath}$ -6$\hat{\jmath}$ $- \lambda\hat{k}$, $\vec{b} = -\hat{\imath}$ +4$\hat{\jmath}$+3$\hat{k}$ and $\vec{c} = -8\hat{\imath}$ -$\hat{\jmath}$+3$\hat{k}$ are coplanar , find λ

37. Find the equation of the line which passes throught the point (1,2,3) and is parallel to the vector $3\hat{\imath}$ +2$\hat{\jmath}$-2$\hat{k}$ both in vector form and Cartesian form.

38. Bag I contains 3 red and 4 black balls and while another bag II contains 5 red and 6 block balls. One ball is drawn at random from one of the bag and it is found to be red. Find the probability that it was drawn from bag II ?

PART D

Answer any SIX questions: **$6 \times 5=30$**

39. Consider f : $R_+ \rightarrow [5,\infty)$ given by f (x) = $9x^2+6x-5$, Show that f is invertible with $f^{-1}(y) = \left\{\frac{\sqrt{y+6}-1}{3}\right\}$

40. If $A=\begin{bmatrix} 2 & 0 & 1 \\ 0 & -3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$, verify $A^3 - 3A^2 - 10A + 24I = O$, where O is zerom matrix of order 3 x 3

41. Solve by matrix method: x+y+z=6, x-2y+3z=6, x-y+z=2

42. If $y = \sin^{-1}x$, Show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$

43. A man of height 2 meters walks at a uniform speed of 5 km/hour, away from a lamp post which is 6 meters high . Find the rate at which the length of the his shadow increases.

44. Find the integral of $\frac{1}{\sqrt{x^2+a^2}}$ w.r.t x and hence evaluate $\int \frac{1}{\sqrt{x^2+2x+4}}dx$.

45. Find the area of the region enclosed by the parabola $x^2=4y$ and the line x=4y-2 and the x axis.

46. Derive the equation of the plane in normal form both in the Cartesian and vector form.

47. Find the particular solution of the differential equation

    $\frac{dy}{dx} + \frac{2xy}{1+x^2} = 1$ when y = 0 and x = 1.

48. A die is thrown 6 times , if 'getting an odd number is success' What is the probability of
    i) 5 success ?
    ii) at least 5 success?
    iii) at most 5 success?.

PART E

Answer any ONE question: **$1 \times 10=10$**

49. (a) A furniture dealer deals in only two items –tables and chairs. He has Rs 50,000 to invest and has storage space of at most 60 pieces. A table costs Rs 2500 and a chair Rs 500. He estimates that from the sale of one table , he can make a profit of Rs 250 and that from the sale of one chair a profit of Rs 75. How many tables and chairs he should buy from the available money so as to maximize his total profit assuming that he can sell all the items which he buys.

    (b)Show that $\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix}$ $=2(x+y+z)^3$

50. (a) Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ and evaluate $\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx$

    (b) Find the points of discontinuity of the function

    $$f(x) = x - [x],$$

    where $[x]$ indicates the greatest integer not greater than x. Also write the set of values of x, where the function is continuous.