Model Question Paper -3

I P.U.C MATHEMATICS (35)

Time : 3 hours 15 minute **Max. Mark: 100**

Instructions:

(i) The question paper has five parts namely A, B, C, D and E. Answer all the parts.

(ii) **Use the graph sheet for the question on Linear inequalities in PART D**.

PART A

Answer ALL the questions **10 × 1=10**

1. Define an empty set .
2. If ($\frac{x+1}{2}$, 7) = (6 , 7) find x .
3. Convert ($\frac{7\pi}{6}$)^c into degrees .
4. Find the real number x if (x - 2i) (1 + i) is purely imaginary .
5. Given 5 flags of different colours how many different signals can be made if each signal requires the use of 2 flags , one below the other .
6. For what value of x the numbers $\frac{-2}{7}$, x , $\frac{-7}{2}$ are in G. P.
7. Find the slope of the line $\frac{x}{3} + \frac{y}{2} = 1$.
8. Find the derivative of $x^2 - 2$ at x = 0 .
9. Write the negation of 'For all $a, b \in I$, $a - b \in I$'.
10. Define sure event .

PART – B

Answer any TEN questions **10 × 2 = 20**

11. In a school , there are 20 teachers who teach mathematics or physics . Of these 12 teach mathematics and 4 teach both physics and mathematics. How many teach physics.
12. If A = { 1 , 2 } , form the set A x A x A .
13. Taking the set of natural numbers as the universal set. If $A = \{x : x \in N, \text{ and } 2x + 1 > 10\}$ and $B = \{x : x \in N, \text{ and } 3x - 1 > 8\}$ find A' and B' .
14. Find the value of cos (- 1710^0) .
15. Prove that sin 2x = $\frac{2\tan x}{1+\tan^2 x}$
16. Find the least positive integer m such that $\left(\frac{1+i}{1-i}\right)^{4m} = 1$.
17. Solve { 3 (2x – 5) - 7 } ≥ 9 (x – 5) .

18. Find the distance of a point (3 , -5) from the line $3x - 4y - 5 = 0$.

19. Find the angle between the lines $y - \sqrt{3}\,x - 5 = 0$ and $\sqrt{3}\,y - x + 6 = 0$.

20. Evaluate $\lim_{x \to -2} \dfrac{\frac{2}{x} + \frac{1}{2}}{x + 2}$

21. Show that the points P(-2 ,3 ,5) , Q (1, 2,3) and R (7 ,0 ,-1) are collinear .

22. Write the converse and contrapositive of 'If a parallelogram is a square, then it is a rhombus'.

23. Write the mean of the given data 6 , 7 , 10 , 12 , 13 , 4 , 6 , 12 .

24. Given P(A) = $\frac{3}{5}$ and P (B) = $\frac{1}{5}$ find P (A or B)

PART – C

Answer any TEN questions **$10 \times 3 = 30$**

25. There are 200 individuals with a skin disorder. 120 has been exposed to the chemical A, 50 to chemical B and 30 to both chemical A and B. Find the number of individuals exposed to

    i) chemical A but not to chemical B ii)Chemical A or chemical B

26. Let A={1,2} , B={1,2,3,4} , and C={5,6} . Verify that $A \times (B \cap C) = (A \times B) \cap (A \times C)$

27. Prove that in any triangle ABC, $\frac{sinA}{a} = \frac{sinB}{b} = \frac{sinC}{c}$

28. Express $\frac{-1+i}{\sqrt{2}}$ in the polar form

29. Solve the equation $x^2 + \frac{x}{\sqrt{2}} + 2 = 0$

30. In how many ways can 5 girls and 3 boys be seated in a row so that no two boys are together.

31. Find the middle term in the expansion of $\left(\frac{x}{3} + 9y\right)^{10}$

32. The number of bacteria in a certain time double every hour. If there are 30 bacteria present in the culture originally. How many bacteria will be present at the end of 2nd hour, 4 th hour, and nth hour.

33. The difference between any two consecutive interior angles of a polygon is 5^0. If the smallest angle is 120^0. Find the number of sides of the polygon.

34. Find the equation of the ellipse whose center at origin , major axis on the X axis and passes through the point (4, 3) and (6,2).

35. Find the derivative of $\tan x$ with respect to x from first principle .

36. Verify by the method of contradiction that $\sqrt{2}$ is irrational

37. One card is drawn from a well shuffled deck of 52 cards. If each outcome is equally likely, calculate the probability that the card will be i) diamond ii)not an ace iii)a black card.

38. A fair coin 1 marked on one face and 6 on the other and a fair die are both tossed . Find the probability that the sum of numbers that turn up is i) 3 ii)12.

PART D

Answer any SIX questions **6 × 5=30**

39. Define modulus function. Draw the graph of modulus function, write down its domain and range .

40. Prove that $\cos^2 2x - \cos^2 6x = \sin 4x \cdot \sin 8x$

41. Prove by mathematical induction that $\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \cdots \ldots\ldots\ldots\ldots \frac{1}{(3n-1)(3n+2)} = \frac{n}{6n+4} \quad \forall\, n \in N.$

42. Solve the following system of inequalities graphically :

    $5x + 4y \leq 40, \qquad x \geq 2, y \geq 3;$

43. What is the number of ways choosing four cards from a pack of 52 playing cards . In how many of these
    - i) Four cards of the same suit
    - ii) are face cards
    - iii) two red and two black card
    - iv) cards are of the same colour.

44. For all real numbers a, b and positive integer 'n' prove that,

    $$(a+b)^n = {}^nC_0\, a^n + {}^nC_1\, a^{n-1}b + {}^nC_2\, a^{n-2}\, b^2 + \ldots + {}^n C_{n-1}\, ab^{n-1} + {}^nC_n\, b^n \ .$$

    Hence prove that $C_0 + C_1 + C_2 + \cdots \ldots\ldots\ldots\ldots C_n = 2^n$

45. Derive a formula for the perpendicular distance of a point (x_1, y_1) from the line Ax+By+C=0.

46. Derive the section formula in 3-D for internal division. Also find the coordinates of the midpoint of the line joining the points $A(1,-2,3)$ and $B(3,4,8)$.

47. Prove that $\lim_{\theta \to 0} \frac{\sin x}{x} = 1$ (x being in radians) and hence evaluate $\lim_{x \to 0} \frac{\sin ax}{\sin bx}$.

48. The mean and standard deviation of 20 observations are found to be 10 and 2 respectively. On rechecking it was found that on observation 8 was incorrect. Calculate the correct mean and the standard deviation in each of the following cases

    i) if wrong item is omitted ii)if it is replaced by 12

PART–E

Answer any ONE question **1 × 10=10**

49. (a) Prove geometrically that $\cos(A+B) = \cos A \cos B - \sin A \sin B$. 6

    (b) Find the derivative of $f(x)=2x^2+3x-5$, also prove that $f'(0) + 3f'(-1) = 0$ 4

50. (a) Define parabola as a set of all points in the plane and derive its equation in the form $y^2 = 4ax, a > 0$ and hence also find the focus and vertex. 6

    (b) Find the sum to 'n' terms of the series $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \cdots$ 4